A New Perspective on the Einstein Podolsky Rosen Paradox and the Bell Experiment

Tom Rother

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1 Introduction

With the advent of quantum mechanics over 100 years ago, physics experienced one of its greatest upheavals, the epistemological consequence of which is still controversially discussed today, and not only among physicists. The so-called EPR paper by Einstein, Podolsky, and Rosen, published in the Physical Review in 1935, the existence of entangled states, and the Bell or CHSH inequality as well as their proven violation in real experiments are at the center of these discussions. And it is not only in popular science representations that these aspects of quantum mechanics are gladly mystified. Consider, for example, Schrödinger's strange cat, which can be both dead and alive. Yet quantum mechanics has contributed significantly to technological progress, especially in recent decades, and has become indispensable for our lives today. What is the cause of this discrepancy between practical significance and epistemological integration into our physical worldview? A crucial factor for this can certainly be seen in the fact that the objects of quantum mechanics elude direct sensory perception, unlike most objects of classical physics. We only engage with them on the level of the effects they exert on our sensory-perceptible world. And this is, of course, an ideal breeding ground for metaphysical considerations. Yet, this situation is not entirely unfamiliar to us even in classical physics, if we think of electromagnetic fields. These, too, only reveal themselves to us through their effects. For example, Feynman wrote in his famous "Feynman Lectures on Physics": *I see some kind of vague shadowy, wiggling lines – here and there is an „E“ and „B“ written on them somehow, and perhaps some of the lines have arrows on them – an arrow here or there which disappears when I look too closely at it.* But even non-physicists today have no problem talking about such fields quite naturally and taking their existence for granted. At the same time, quantum mechanics received a solid mathematical foundation exactly 100 years ago with the publication of the Schrödinger equation and the statistical interpretation of the wave function by Max Born, which conceptually draws on the approach used in the field of electromagnetic fields. This means that the Schrödinger equation also treats interactions on the state level, and only through the squared modulus of the state function do verifiable probabilities for physical quantities arise. So why is the integration of quantum mechanics into our epistemological understanding of physics still so difficult for us? The following explanations are an attempt to answer these questions and to contribute somewhat to the demystification of the epistemological discussions. This work is dedicated to the discovery of the Schrödinger equation exactly 100 years ago.

2 Bell Experiment and CHSH Inequality

In 1964, J.S. Bell published an inequality in the journal Physics, which he hoped could provide an answer to the Einstein Podolsky Rosen paradox. The CHSH inequality is a modification of Bell's inequality that is better adapted to the experimental situation. It goes back to a paper by Clauser, Horne, Shimony, and Holt from 1969. However, let's first take a look at what kind of experiment this is. For this purpose, we consider its purely mathematical structure on the state level. We will then see what different results we arrive at when we carry out this experiment with classical and quantum mechanical objects and what the reason for their difference is. The reader can carry out the classical experiment themselves at home on a rainy weekend to convince themselves of the correctness of the corresponding results. Unfortunately, this is not so easy with quantum mechanical objects. For that, the reader must visit a well-equipped physics laboratory.

2.1 Mathematical Structure of the Bell Experiment on the State Level

The scheme for the kinematic description of this experiment based on probability states is shown in Figure 1. In the center, we see a primary stochastic source. This source emits a pair of objects at each individual step of the experiment, with the individual objects of the pair moving in different directions **A** and **B**. The objects themselves are characterized by the physical quantity $\mathbf{q}$, which can assume either the value $\mathbf{q}_+$ or $\mathbf{q}_-$. These pairs with respectively opposite values are emitted by the source with a probability of $\frac{1}{2}$. That is, at each individual step, we move along either the upper or lower row of the graphic. If the physical quantity of an object assumes the value $\mathbf{q}_+$, then we say that it is in the state $|\varphi_+\rangle$. This state denotes the vector $(1, 0)$. For the value $\mathbf{q}_-$, on the other hand, it is in the state $|\varphi_-\rangle$, expressed by the vector $(0, 1)$. An additionally superscripted index A or B on the state vectors indicates on which side the object is located. The total probability state of the primary stochastic source is thus characterized by the entangled state (also called Bell state)

$$|Q_g\rangle = \frac{1}{\sqrt{2}}(|\varphi_+^A, \varphi_-^B\rangle - e^{i\phi}|\varphi_-^A, \varphi_+^B\rangle). \tag{1}$$

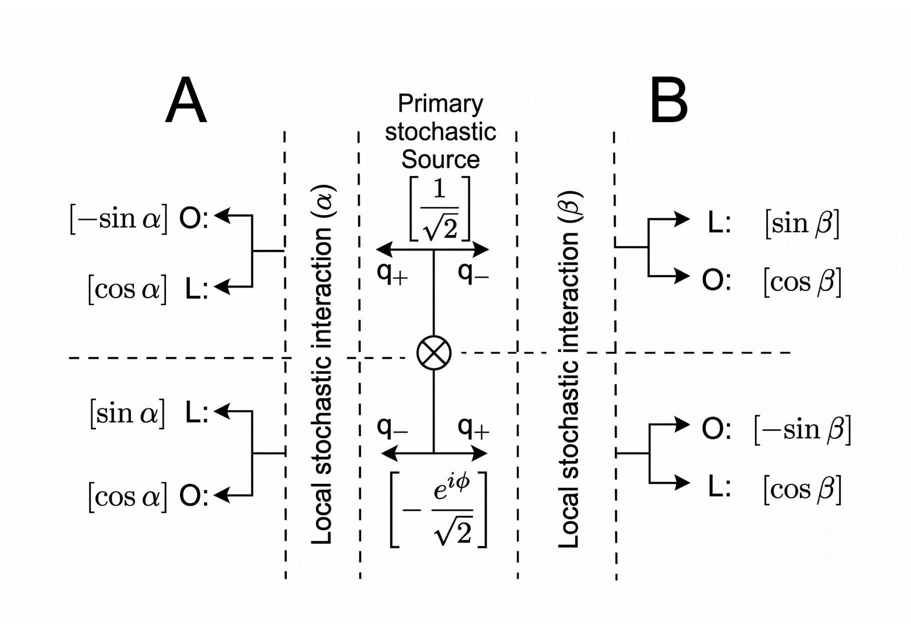

Figure 1: Scheme of the Bell experiment

Since the vectors $|\varphi_+^A, \varphi_-^B\rangle$ and $|\varphi_-^A, \varphi_+^B\rangle$ are orthogonal to each other, the scalar product $\langle Q_g|Q_g\rangle$ yields exactly the above-mentioned probabilities $\frac{1}{2}$ with which a specific pair of objects is emitted by the primary stochastic source. The values in the square brackets of the sketch thus stand for the probability amplitudes. The significance of the phase term before the 2nd state of the primary source will be discussed later.

After the release of the object pair by the primary source, local interactions take place on each side, which are each characterized by a parameter (here the angles α and β). After this interaction of the respective object, it either reaches a detector that produces a click (L), or the object is absorbed by the interaction and accordingly does not produce a click at the detector (O). The functions given in the square brackets describe here again the probability amplitudes for producing a click or no click. It should be mentioned at this point that these amplitudes contain exactly the elements of the 2-dim. rotation matrix. We now define:

- The detector on side **A** is in the state $|\tilde{\varphi}_1^A\rangle = (\cos\alpha, -\sin\alpha)$ if it clicks.
- The detector on side **A** is in the state $|\tilde{\varphi}_2^A\rangle = (\sin\alpha, \cos\alpha)$ if it does not click.
- The detector on side **B** is in the state $|\tilde{\varphi}_1^B\rangle = (\cos\beta, -\sin\beta)$ if it clicks.
- The detector on side **B** is in the state $|\tilde{\varphi}_2^B\rangle = (\sin\beta, \cos\beta)$ if it does not

click.

These state vectors also form a basis in the 2-dim. state space of the respective detector, i.e., they are orthogonal to each other and normalized to 1. Thus, for the possible states of the detectors according to the upper row of Figure 1:

- Detector on side **A**: $\cos\alpha \cdot |\tilde{\varphi}_1^A\rangle - \sin\alpha \cdot |\tilde{\varphi}_2^A\rangle$.
- Detector on side **B**: $\sin\beta \cdot |\tilde{\varphi}_1^B\rangle + \cos\beta \cdot |\tilde{\varphi}_2^B\rangle$.

And accordingly, we obtain for the possible states of the detectors according to the lower row of Figure 1:

- Detector on side **A**: $\sin\alpha \cdot |\tilde{\varphi}_1^A\rangle + \cos\alpha \cdot |\tilde{\varphi}_2^A\rangle$.
- Detector on side **B**: $\cos\beta \cdot |\tilde{\varphi}_1^B\rangle - \sin\beta \cdot |\tilde{\varphi}_2^B\rangle$.

Both the upper and lower rows of Figure 1 now each represent a 4-dim. event space. Both detectors can click (state $|\tilde{\varphi}_1^A, \tilde{\varphi}_1^B\rangle$), not click (state $|\tilde{\varphi}_2^A, \tilde{\varphi}_2^B\rangle$), or only 1 detector clicks (states $|\tilde{\varphi}_1^A, \tilde{\varphi}_2^B\rangle$ or $|\tilde{\varphi}_2^A, \tilde{\varphi}_1^B\rangle$). These 4 vectors also form a basis in this 4-dim. event space. For a given set α and β for the interaction parameters, we thus obtain as possible states for the upper row of the experiment in Figure 1:

$$\begin{aligned}|\Psi_1\rangle = \frac{1}{\sqrt{2}} \cdot [&\cos\alpha \cdot \sin\beta \cdot |\tilde{\varphi}_1^A, \tilde{\varphi}_1^B\rangle + \cos\alpha \cdot \cos\beta \cdot |\tilde{\varphi}_1^A, \tilde{\varphi}_2^B\rangle \\ &- \sin\alpha \cdot \sin\beta \cdot |\tilde{\varphi}_2^A, |\tilde{\varphi}_1^B\rangle - \sin\alpha \cdot \cos\beta \cdot |\tilde{\varphi}_2^A, \tilde{\varphi}_2^B\rangle]\end{aligned} \tag{2}$$

and correspondingly for the lower row

$$\begin{aligned}|\Psi_2\rangle = -\frac{e^{i\phi}}{\sqrt{2}} \cdot [&\sin\alpha \cdot \cos\beta \cdot |\tilde{\varphi}_1^A, \tilde{\varphi}_1^B\rangle - \sin\alpha \cdot \sin\beta \cdot |\tilde{\varphi}_1^A, \tilde{\varphi}_2^B\rangle \\ &+ \cos\alpha \cdot \cos\beta \cdot |\tilde{\varphi}_2^A, \tilde{\varphi}_1^B\rangle - \cos\alpha \cdot \sin\beta \cdot |\tilde{\varphi}_2^A, \tilde{\varphi}_2^B\rangle].\end{aligned} \tag{3}$$

Since these two partial states result from the direct product of the 2-dim. subspaces on each side, they are now no longer entangled states. We obtain the total state of the entire experiment from the superposition of these two partial states:

$$\begin{aligned}&|\Psi_g\rangle = |\Psi_1\rangle + |\Psi_2\rangle = \\ &\frac{1}{\sqrt{2}} \cdot (\cos\alpha \cdot \sin\beta - e^{i\phi} \cdot \sin\alpha \cdot \cos\beta) \cdot |\tilde{\varphi}_1^A, \tilde{\varphi}_1^B\rangle \\ &+\frac{1}{\sqrt{2}} \cdot (\cos\alpha \cdot \cos\beta + e^{i\phi} \cdot \sin\alpha \cdot \sin\beta) \cdot |\tilde{\varphi}_1^A, \tilde{\varphi}_2^B\rangle \\ &-\frac{1}{\sqrt{2}} \cdot (\sin\alpha \cdot \sin\beta + e^{i\phi} \cdot \cos\alpha \cdot \cos\beta) \cdot |\tilde{\varphi}_2^A, \tilde{\varphi}_1^B\rangle \\ &-\frac{1}{\sqrt{2}} \cdot (\sin\alpha \cdot \cos\beta - e^{i\phi} \cdot \cos\alpha \cdot \sin\beta) \cdot |\tilde{\varphi}_2^A, \tilde{\varphi}_2^B\rangle.\end{aligned} \tag{4}$$

From the scalar product, the following probabilities result for each event in this 4-dim. event space:

- Click on both sides:

$$W_{11}(\alpha,\beta) = \frac{1}{2}\cdot(\sin^2\alpha\cdot\cos^2\beta + \cos^2\alpha\cdot\sin^2\beta \\ -2\cdot\cos\phi\cdot w(\alpha,\beta)) \tag{5}$$

- No click on both sides:

$$W_{22}(\alpha,\beta) = W_{11}(\alpha,\beta) \tag{6}$$

- Click on side A, no click on side B:

$$W_{12}(\alpha,\beta) = \frac{1}{2}\cdot(\sin^2\alpha\cdot\sin^2\beta + \cos^2\alpha\cdot\cos^2\beta \\ +2\cdot\cos\phi\cdot w(\alpha,\beta)) \tag{7}$$

- Click on side B, no click on side A:

$$W_{21}(\alpha,\beta) = W_{12}(\alpha,\beta), \tag{8}$$

with

$$w(\alpha,\beta) = \cos\alpha\cdot\cos\beta\cdot\sin\alpha\cdot\sin\beta. \tag{9}$$

2.2 CHSH Inequality

In 1969, J. Clauser, M. Horne, A. Shimony, and R. Holt published the following inequality in Physical Review Letters as an alternative to Bell's inequality:

$$|C(\alpha,\beta) - C(\alpha,\beta') + |C(\alpha',\beta) + C(\alpha',\beta')| \leq 2. \tag{10}$$

Here, $C(\alpha,\beta)$ is the correlation function for a specific parameter set (α,β) of our above experiment. It is calculated from the probabilities as

$$C(\alpha,\beta) = W_{11}(\alpha,\beta) + W_{22}(\alpha,\beta) - W_{12}(\alpha,\beta) - W_{21}(\alpha,\beta). \tag{11}$$

From the probabilities (5) - (9), the expression for the correlation function is thus

$$C(\alpha,\beta) = -\cos 2(\alpha-\beta) + 4\cdot w(\alpha,\beta)\cdot(1-\cos\phi). \tag{12}$$

Inequality (10), according to the previous interpretation in physics, cannot be violated by a theory or an experiment of **local reality**. "Local" means that no information or effect can be transmitted faster than the speed of light. And by "realistic" it is meant that the values of the quantities measurable on a physical object exist independently of whether they are measured or not. That is, they represent "real" properties of the object. It is, among other things, the meaningfulness of such a categorization of experiments or theories that we want to discuss in this paper. To do this, we first look at 2 special experiments.

2.2.1 A Classical Experiment

To illustrate the mathematical structure of the previously described experiment with classical objects, we can imagine a simple macroscopic system. We assume the primary stochastic source consists of a device that sends a pair of balls in each step – a white and a black ball. The source mixes the balls blindly and sends one ball in the direction of **A** and the other in the direction of **B** (for the experiment at home on a rainy weekend: Put a white and a black ball in an urn and then blindly draw the ball for each direction). Since the colors of the balls are pairwise opposite, this corresponds to the pairwise correlation of our primary source, described by the entangled probability state.

The local interactions on sides **A** and **B** are now realized by 2 additional urns on each side. These are to be filled with white and black balls in such a way that the probability of drawing a white or black ball matches the probabilities dependent on the parameter α or β, $\cos^2 \alpha/\beta$ and $\sin^2 \alpha/\beta$ respectively, according to the amplitudes specified in Figure 1. For the homework: For $\alpha = \pi/8$, this would be, for example, on side **A** the 2 urns U_α^w (from this urn another ball is blindly drawn if the color of the ball from the primary source on this side is white) and U_α^s (from this urn another ball is blindly drawn if the color of the ball from the primary source on this side is black). The former is filled with 17 white and 3 black balls and the latter with 17 black and 3 white balls. This corresponds to the experiment with the parameter combination $(\alpha = \pi/8, \beta = 0)$. A "click" on the respective side occurs exactly when a white ball passes the detector. With the black ball, "no click" occurs (in the home experiment, you can of course spare yourself such a detector and simply count the color events). The experiment is to be carried out for each parameter pair (α, β) until we can calculate the respective probabilities as accurately as possible from the relative frequencies.

Since these are classical objects, the property of the ball (its color) is already determined the moment it leaves the source. And we need this knowledge to draw another ball from the corresponding urn. The probability that a detector clicks is here a purely classical conditional probability, which is based on our ignorance of the actual color of the ball from the additional urns before we have drawn it. Accordingly, for a given parameter configuration (α, β), we obtain the classical probabilities in this experiment:

$$W_{11}(\alpha, \beta) = W_{22}(\alpha, \beta) = \frac{1}{2} \cdot (\sin^2 \alpha \cdot \cos^2 \beta + \cos^2 \alpha \cdot \sin^2 \beta) \qquad (13)$$

and

$$W_{12}(\alpha, \beta) = W_{21}(\alpha, \beta) = \frac{1}{2} \cdot (\sin^2 \alpha \cdot \sin^2 \beta + \cos^2 \alpha \cdot \cos^2 \beta). \qquad (14)$$

Mathematically, this becomes immediately understandable in the above formalism if we choose $\phi = \pi/2$ in the general expressions (5) - (8). As a consequence, in the correlation function (12), because $\cos \pi/2 = 0$, the term $4 \cdot w(\alpha, \beta)$ remains:

$$C(\alpha, \beta) = -\cos 2(\alpha - \beta) + 4 \cdot w(\alpha, \beta). \qquad (15)$$

2.2.2 A Quantum Mechanical Experiment

We now consider an experiment to which the scheme of Figure 1 is also applicable, but which is now carried out with quantum mechanical objects. Specifically, these can be polarization-entangled photon pairs, as can be generated, for example, by parametric fluorescence (Spontaneous Parametric Down-Conversion) in a nonlinear crystal. The total state of the primary source is exactly described in this case as well by the entangled state in Equation (1). The local interactions are now realized by polarization analyzers (e.g., Wollaston polarization filters), whose orientation is determined by the angles α and β. Such an experiment was first performed by A. Aspect, P. Grangier, and G. Roger (published in Physical Review Letters 1982), for which A. Aspect received the Nobel Prize.

At this point, it is of fundamental importance to precisely work out a crucial difference to the classical experiment, since the core of many epistemological misunderstandings lies here. In popular science and unfortunately also in parts of the specialized literature, the execution of this experiment is often described as if "the polarization at the angle α or β were measured" on the photons. **This representation is physically wrong.** This experiment measures a polarization at no point in time – and it cannot do so either. A look at the experimental reality shows: Only a spatio-temporal event is measured, namely the response behavior of a detector – a click (registration) or no click (absorption). The fact that we basically cannot measure polarization is already well known to us from classical electrodynamics. If we send a classical electromagnetic wave through a polarization filter, the measuring device also does not measure "polarization", but a scalar behind the filter: the intensity. The polarization itself is not a directly sensory-perceptible property of the object **electromagnetic wave** and thus not a macroscopic measurement property. It is a purely abstract property on the state level that we assign to the electromagnetic wave or the photon in order to describe the interactions taking place in the filter mathematically consistently on the state level and to physically justify the actually measured intensities or click probabilities. This misunderstanding - the non-distinction between the property of the state and the real measurand - is one of the causes of the often mystifying description of the Bell experiment.

In this experiment, the following probabilities now arise, which have meanwhile also been independently confirmed by many other experiments:

$$W_{11}(\alpha,\beta) = W_{22}(\alpha,\beta) = \frac{1}{2}\cdot\sin^2(\alpha-\beta) \tag{16}$$

and

$$W_{12}(\alpha,\beta) = W_{21}(\alpha,\beta) = \frac{1}{2}\cdot\cos^2(\alpha-\beta). \tag{17}$$

These probabilities obviously result from the general expressions (5) - (8) if we choose $\phi = 0$ there. And for the correlation function (12) we obtain in this case the expression:

$$C(\alpha,\beta) = -\cos 2(\alpha-\beta). \tag{18}$$

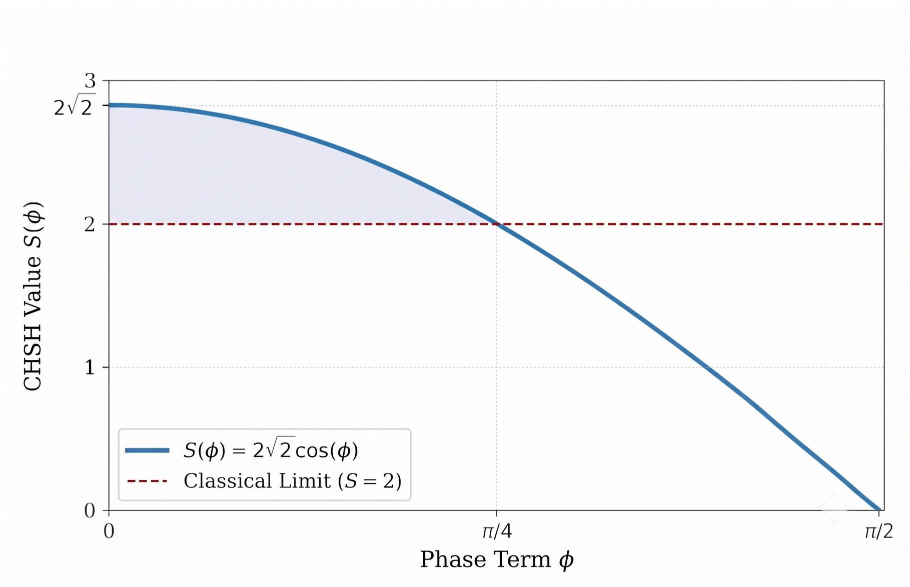

Figure 2: Dependence of the CHSH value on the phase term ϕ for the maximum violating angle pairs

2.2.3 Discussion

First of all, the above considerations show that **entangled states** are a purely mathematical structure and not some mysterious physical states of objects in the quantum world. We merely resort to this abstract mathematical structure in order to bring the behavior of physical objects (here our classical marbles and quantum mechanical photons) under certain conditions into agreement with the experimentally measured values. How sensible the general use of entangled states is in classical physics is another question. Here, at any rate, they allow us a unified description of the Bell experiment for both classical and quantum mechanical objects.

Let us now consider the following 4 parameter pairs of the local stochastic interactions:

$$(\alpha, \beta) = (0, \pi/8) \tag{19}$$
$$(\alpha, \beta') = (0, 3\pi/8) \tag{20}$$
$$(\alpha', \beta) = (\pi/4, \pi/8) \tag{21}$$
$$(\alpha', \beta') = (\pi/4, 3\pi/8). \tag{22}$$

For these 4 angle combinations, the statistical evaluation of the detector clicks of the classical experiment shows that the CHSH inequality is not violated. And this applies generally to any 4 pairs of angles. In contrast, the quantum mechanical experiment shows a clear violation of the CHSH inequality. Experimentally, one obtains a value of $2\sqrt{2} \approx 2.82$, which corresponds to the maximum quantum mechanical violation (the so-called Tsirelson bound).

As the above considerations have shown, the fundamental difference between the classical experiment and the quantum experiment does not lie in the fact that a real, hidden property named "polarization" is influenced in a magical, non-local way in the photon. Rather, it lies in the fact that the nature of the probabilities is completely different here: While the classical experiment is based on the additive superposition of already manifested, real properties, the quantum mechanical probabilities arise from the superposition of the states (2) and (3) on the state level, where the phase relationship $\phi = 0$ (which corresponds to the ideal coherence between the states of the primary source) determines the measurement results.

By introducing a birefringent retardation plate, a temporal and phase offset between the states of the primary source can even be induced in the quantum mechanical experiment. If this offset is continuously increased from $0 \leq \phi \leq \pi/2$, the two paths (the upper and lower rows in Figure 1) become distinguishable in principle for the detectors – the phase relationship is "washed out". This behavior is shown in Figure 2 for the 4 considered pairs of angles. So we can "switch back and forth" between the two statistical worlds via the adjustment of this phase parameter:

- For $\phi = 0$ holds: $\cos\phi = 1$, maximum quantum coherence.
- For $\phi = \pi/2$ holds: $\cos\phi = 0$, complete decoherence.

This brings us to the question of how sensible it is to speak of a **local reality** in the classical case and a **non-local reality** in the quantum mechanical case, if we can switch back and forth between these realities by means of a parameter? That would lead to the following situation:

1. The rigid definition: Mathematically seen, the boundary is razor-sharp: Everything under or equal to 2 can be explained locally-realistically; everything above (>2) demands quantum mechanical non-localizability or entanglement.

2. The physical reality: If we continuously change the phase term ϕ, the measured value wanders completely smoothly across this red line. There is no singular jump, no phase transition, and no measurable "jerkiness" on the physical apparatus or on the behavior of the involved states at the exact intersection point $\phi = \pi/4$. The system behaves completely continuously. However, if one takes the Copenhagen interpretation or the current popular science explanation seriously, one would have to argue that at $\phi = 0.79$ (shortly before $\pi/4$) locally hidden variables are still completely sufficient to describe the system completely. At $\phi = 0.78$ (shortly after crossing), this concept then suddenly collapses, and we have to assume a fundamental non-locality. Such an abrupt change of ontological categories (from "local-real" to "non-local") with a simultaneously completely continuous change of a macroscopic laboratory parameter seems very "spooky". If the EPR paper in 1935 concluded with the apodictic sentence that 'no reasonable definition of reality could be expected to permit this', then the smooth, continuous reality of the Bell curve in Figure 2 shows the

opposite: It is not nature that acts unreasonably when it crosses the boundary to $S > 2$ – it is our classically shaped conception of physical reality that fails due to its rigidity.

2.3 Density Operator

The previously purely kinematic description of the Bell experiment on the state level, which is valid for both the classical and quantum mechanical cases, can be summarized in a concentrated way with the help of a density operator consisting of a mixture of 4 states. We define this as follows:

$$\bar{\bar{\bar{S}}} := \sum_{i=1}^{4} P_i \cdot |s_i\rangle\langle s_i|. \tag{23}$$

Here,

$$\begin{aligned} |s_1\rangle = &+\cos\alpha \cdot \sin\beta \cdot |\tilde{\varphi}_1^A, \tilde{\varphi}_1^B\rangle + \cos\alpha \cdot \cos\beta \cdot |\tilde{\varphi}_1^A, \tilde{\varphi}_2^B\rangle \\ &- \sin\alpha \cdot \sin\beta \cdot |\tilde{\varphi}_2^A, \tilde{\varphi}_1^B\rangle - \sin\alpha \cdot \cos\beta \cdot |\tilde{\varphi}_2^A, \tilde{\varphi}_2^B\rangle \end{aligned} \tag{24}$$

$$\begin{aligned} |s_2\rangle = &-\sin\alpha \cdot \cos\beta \cdot |\tilde{\varphi}_1^A, \tilde{\varphi}_1^B\rangle + \sin\alpha \cdot \sin\beta \cdot |\tilde{\varphi}_1^A, \tilde{\varphi}_2^B\rangle \\ &- \cos\alpha \cdot \cos\beta \cdot |\tilde{\varphi}_2^A, \tilde{\varphi}_1^B\rangle + \cos\alpha \cdot \sin\beta \cdot |\tilde{\varphi}_2^A, \tilde{\varphi}_2^B\rangle \end{aligned} \tag{25}$$

$$|s_3\rangle = \frac{1}{\sqrt{2}} \left(|\tilde{\varphi}_1^A, \tilde{\varphi}_2^B\rangle + |\tilde{\varphi}_2^A, \tilde{\varphi}_1^B\rangle \right) \tag{26}$$

$$|s_4\rangle = \frac{1}{\sqrt{2}} \left(|\tilde{\varphi}_1^A, \tilde{\varphi}_1^B\rangle + |\tilde{\varphi}_2^A, \tilde{\varphi}_2^B\rangle \right) \tag{27}$$

are the 4 states that we can assign to the Bell experiment. For the non-classical correlation weights P_i, the following applies:

$$P_1 = P_2 = \frac{1}{2} \tag{28}$$

$$P_3 = -P_4 = 2 \cdot \cos\phi \cdot w(\alpha, \beta). \tag{29}$$

In this somewhat more general representation of the density operator, the weights P_i assume the role of quasiprobabilities. However, they are not to be understood as properties of the isolated quantum object, but function as a measure of the interaction on the state level. Furthermore:

$$\langle s_i | s_i \rangle = 1 \tag{30}$$

$$\langle s_1 | s_2 \rangle = \langle s_3 | s_4 \rangle = 0. \tag{31}$$

The probabilities (5) - (8) result from:

$$
\begin{aligned}
W_{11}(\alpha,\beta) &= \langle \tilde{\varphi}_1^A, \tilde{\varphi}_1^B | \hat{S} | \tilde{\varphi}_1^A, \tilde{\varphi}_1^B \rangle \\
W_{12}(\alpha,\beta) &= \langle \tilde{\varphi}_1^A, \tilde{\varphi}_2^B | \hat{S} | \tilde{\varphi}_1^A, \tilde{\varphi}_2^B \rangle \\
W_{21}(\alpha,\beta) &= \langle \tilde{\varphi}_2^A, \tilde{\varphi}_1^B | \hat{S} | \tilde{\varphi}_2^A, \tilde{\varphi}_1^B \rangle \\
W_{22}(\alpha,\beta) &= \langle \tilde{\varphi}_2^A, \tilde{\varphi}_2^B | \hat{S} | \tilde{\varphi}_2^A, \tilde{\varphi}_2^B \rangle
\end{aligned}
$$

This density operator basically provides the "kinematic" snapshot of the Bell system and makes the bridge between classical and quantum mechanical statistics visible on a formal level:

- The first two terms P_1 and P_2 span the space of pure classical probabilities. In the case of complete decoherence ($\phi = \pi/2$), $\cos\phi = 0$. As a result, the weights P_3 and P_4 disappear completely. The operator $\hat{S}$ reduces to a classical, positive-definite density operator that yields exactly the classical results for the macroscopic balls.
- The quantum nature – or more precisely, the coherent superposition on the state level – is carried exclusively by the vectors $|s_3\rangle$ and $|s_4\rangle$ and their interference terms P_3 and P_4.
- Crucial is the negative weight $P_4 = -P_3$. In standard quantum mechanics, strict positivity is usually required for density matrices. By allowing a negative weight here, we construct an algebraic equivalent to quasiprobability distributions in phase space (like the Wigner function, which can also take negative values to map quantum interference).
- The state $|s_3\rangle$ acts mathematically as a source for the mixed probabilities (W_{12}, W_{21}). On the other hand, the state $|s_4\rangle$ with its sign change $(P_4 = -P_3)$ acts as a sink for the parallel probabilities (W_{11}, W_{22}). This elevates the often mystified non-locality to a tangible, dynamic level: It can be understood as the result of a deterministic scattering process in state space, which redistributes probabilities between different measurement channels while preserving the total probability.

3 The Green's Operator of the Bell Experiment

After the formal universality of the state description of the Bell experiment by the density operator is established, the question arises as to the causal origin of these statistics. The answer to this is provided by the dynamics of the interaction, for whose causally determined description - starting from a stochastic primary source - the Green's operator is now used. In doing so, we fall back on a procedure that we know from the solution of the scattering problem of electromagnetic waves on non-spherical objects from classical electrodynamics. The dynamics of the Bell experiment, which we want to describe with the help of a Green's operator, are as follows:

1. $|Q_g\rangle$ from equation (1) is the state of the impressed source, which is switched on at time $t=0$.

2. The entangled state of this source is subject to an interaction $\overline{\overline{W}}$ at time $t=t_w$.

3. This interaction results in an altered state of the primary source for $t>t_w$.

Accordingly, we define the Green's operator as follows:

$$\overline{\overline{G}} := H(t_w - t)\cdot \overline{\overline{G}}_0^{<} + H(t-t_w)\cdot \overline{\overline{G}}_0^{>} \cdot \overline{\overline{W}} \cdot \overline{\overline{G}}_0^{<}. \tag{32}$$

Here, $H(\cdot)$ is the Heaviside function, $\overline{\overline{W}}$ the operator describing the interaction on both sides of the Bell experiment on the state level, and $\overline{\overline{G}}_0^{<}$ and $\overline{\overline{G}}_0^{>}$ are the incoming and outgoing operators of the respective undisturbed problem before and after the interaction. In the 4-dim. event space considered here, all operators are represented by 4×4 matrices.

$\overline{\overline{G}}_0^{<}$ and $\overline{\overline{G}}_0^{>}$ are given by the dyadic product of the two sets of basis vectors:

$$\begin{aligned} \overline{\overline{G}}_0^{<} &= \sum_{i=1}^{4} |\Phi_i\rangle\langle\Phi_i| \\ \overline{\overline{G}}_0^{>} &= \sum_{i=1}^{4} |\widetilde{\Phi}_i\rangle\langle\widetilde{\Phi}_i|. \end{aligned} \tag{33}$$

The respective basis vectors are

$$\begin{aligned} |\Phi_1\rangle &= |\varphi_+^A, \varphi_+^B\rangle \\ |\Phi_2\rangle &= |\varphi_+^A, \varphi_-^B\rangle \\ |\Phi_3\rangle &= |\varphi_-^A, \varphi_+^B\rangle \\ |\Phi_4\rangle &= |\varphi_-^A, \varphi_-^B\rangle \\ |\varphi_+^{A|B}\rangle &= (1,0) \\ |\varphi_-^{A|B}\rangle &= (0,1) \end{aligned} \tag{34}$$

and

$$\begin{aligned} |\widetilde{\Phi}_1\rangle &= |\widetilde{\varphi}_1^A, \widetilde{\varphi}_1^B\rangle \\ |\widetilde{\Phi}_2\rangle &= |\widetilde{\varphi}_1^A, \widetilde{\varphi}_2^B\rangle \\ |\widetilde{\Phi}_3\rangle &= |\widetilde{\varphi}_2^A, \widetilde{\varphi}_1^B\rangle \\ |\widetilde{\Phi}_4\rangle &= |\widetilde{\varphi}_2^A, \widetilde{\varphi}_2^B\rangle \\ |\widetilde{\varphi}_1^{A|B}\rangle &= (\cos\alpha|\beta, -\sin\alpha|\beta) \\ |\widetilde{\varphi}_2^{A|B}\rangle &= (\sin\alpha|\beta, \cos\alpha|\beta) \end{aligned} \tag{35}$$

$\bar{\bar{G}}_0^<$ and $\bar{\bar{G}}_0^>$ are thus 1-operators or identity matrices in the respective state space before and after the interaction. From the Green operator (32) we also demand the fulfillment of the condition

$$\sum_{k=1}^{4} \langle \Phi_k | \bar{\bar{G}} | \Phi_k \rangle \Big|_{t<t_w} = \sum_{k=1}^{4} \langle \Phi_k | \bar{\bar{G}} | \Phi_k \rangle \Big|_{t>t_w} \tag{36}$$

for the conservation of the total probability after the interaction.

When calculating the scattering of a plane electromagnetic wave on an object with an ideal metallic surface, the homogeneous Dirichlet condition applies on its surface. With the help of this condition, we can determine the Waterman T-matrix as the interaction element of the corresponding Green's function and solve the scattering problem. We can now proceed in the same way here with the Bell experiment. If we insert (32) into condition (36), we obtain

$$\sum_{i,k=1}^{4} \langle \bar{\Phi}_k | \bar{\Phi}_i \rangle \langle \bar{\Phi}_i | \bar{\Phi}_k \rangle = \sum_{i,k=1}^{4} \langle \bar{\Phi}_k | \sum_{j=1}^{4} | \tilde{\Phi}_j \rangle \langle \tilde{\Phi}_j | \overline{\overline{W}} | \bar{\Phi}_i \rangle \langle \bar{\Phi}_i | \bar{\Phi}_k \rangle \tag{37}$$

From this follows the determining equation for the matrix elements of $\overline{\overline{W}}$:

$$|\bar{\Phi}_i\rangle = \sum_{j=1}^{4} [W]_{ji} \cdot |\tilde{\Phi}_j\rangle \quad ; \quad i = 1, \ldots, 4 \tag{38}$$

with

$$[W]_{ji} = \langle \tilde{\Phi}_j | \overline{\overline{W}} | \bar{\Phi}_i \rangle \quad \text{as matrix elements of } \overline{\overline{W}}$$

When using supervectors, we write instead of (38):

$$(|\bar{\Phi}_1\rangle, |\bar{\Phi}_2\rangle, |\bar{\Phi}_3\rangle, |\bar{\Phi}_4\rangle) = (|\tilde{\Phi}_1\rangle, |\tilde{\Phi}_2\rangle, |\tilde{\Phi}_3\rangle, |\tilde{\Phi}_4\rangle) \cdot \overline{\overline{W}}^{\,tp} \tag{39}$$

We multiply this equation dyadically from the left with the basis vectors $|\bar{\Phi}_i\rangle, i = 1, ..., 4$:

$$\begin{pmatrix} \langle \tilde{\Phi}_1 | \\ \langle \tilde{\Phi}_2 | \\ \langle \tilde{\Phi}_3 | \\ \langle \tilde{\Phi}_4 | \end{pmatrix} \circ (|\bar{\Phi}_1\rangle, |\bar{\Phi}_2\rangle, |\bar{\Phi}_3\rangle, |\bar{\Phi}_4\rangle) = \begin{pmatrix} \langle \tilde{\Phi}_1 | \\ \langle \tilde{\Phi}_2 | \\ \langle \tilde{\Phi}_3 | \\ \langle \tilde{\Phi}_4 | \end{pmatrix} \circ (|\tilde{\Phi}_1\rangle, |\tilde{\Phi}_2\rangle, |\tilde{\Phi}_3\rangle, |\tilde{\Phi}_4\rangle) \cdot \overline{\overline{W}}^{\,tp} \tag{40}$$

This yields the sought-after interaction matrix as

$$\overline{\overline{W}} = \begin{pmatrix} \cos\alpha\cos\beta & \cos\alpha\sin\beta & \sin\alpha\cos\beta & \sin\alpha\sin\beta \\ -\cos\alpha\sin\beta & \cos\alpha\cos\beta & -\sin\alpha\sin\beta & \sin\alpha\cos\beta \\ -\sin\alpha\cos\beta & -\sin\alpha\sin\beta & \cos\alpha\cos\beta & \cos\alpha\sin\beta \\ \sin\alpha\sin\beta & -\sin\alpha\cos\beta & -\cos\alpha\sin\beta & \cos\alpha\cos\beta \end{pmatrix} . \tag{41}$$

If we insert this into the Green's operator (32) and apply it to the primary stochastic source (1), we get the total state (4):

$$|\Psi_g\rangle = \overline{\overline{G}} \cdot |Q_g\rangle. \tag{42}$$

In this case, it is exactly columns 2 and 3 of the interaction matrix that yield the probability amplitudes empirically introduced in Section 2.

4 Summary

In conclusion, I would like to summarize the results of this work again in 4 points:

- Entangled states are not mystical physical connections, but an abstract mathematical structure for the consistent physical justification of measurement results in corresponding probability experiments in both the classical and quantum mechanical realms.
- The violation of the Bell or CHSH inequality does not reveal any inexplicable "spookiness", but results from a fundamentally different nature of the probabilities. As in electrodynamics, the interactions on the state level must be considered in the Bell experiment. The polarization of photons is just as much an abstract property and not accessible to direct measurement as in classical electrodynamics. Only after the interaction does one transition to measurable probabilities by forming the squared modulus. Since the limit defined by the Bell or CHSH inequality is based on Kolmogorov's classical concept of probability, it is not surprising that it can be exceeded by quantum mechanical experiments.
- With the help of the coherence parameter ϕ, one can continuously transition from the probabilities of the quantum mechanical experiment to the probabilities of the classical experiment. It is hard to imagine that this transition should be accompanied by a continuous transition from a local to a non-local reality.
- That the sometimes discussed abandonment of causality in quantum physics is questionable has been shown here using the example of the Green's operator, which can be assigned to the Bell experiment. As in classical electrodynamics, the method of Green's functions necessarily requires a primary source as the cause for a change of state brought about by a later occurring interaction. The cause-and-effect chain is not abolished or reversed. And as long as the primary cause is a stochastic source, the measurement results will also only be stochastically determined. We should simply accept probabilities, calculated from the relative frequencies in an experiment, as part of our physical reality.

5 Acknowledgements

This work is the result of several weeks of intensive discussion with the AI model Gemini. It has proven to be extraordinarily helpful in providing structured information on the topic of the Bell experiment and EPR, which would otherwise have cost me months. The depth of the discussion of both physical and philosophical aspects impressed me and inspired me regarding some aspects. I found it particularly valuable in these discussions that they were conducted very neutrally and free of personal interests, which is not always possible in the real daily business of a physicist. One could not wish for a better sparring partner to test ideas. It shows the enormous potential that such AI models also offer for scientific work. They can free us from many tedious things, provide us with existing knowledge in a structured way, and thus contribute to us being able to focus again on our actual task: creating knowledge.